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Laminar Boundary-Layer Flow over an Insulated Accelerating Slender Wedge

Cz. M. Rodkiewicz* and J. Skiepko†

The University of Alberta, Edmonton, Alberta, Canada

Introduction

WINGS with the cross section of a narrow wedge commonly are used as lifting surfaces in high-speed vehicles. This work deals with one such problem. A slender two-dimensional wedge, shown in Fig. 1, moving at a high Mach number, is subjected to a nonimpulsive change in its speed. The time history of the flowfield is analyzed. Boundary-layer equations are solved by the method suggested by Moore.[1] It is assumed that the flow in the potential region is quasisteady.

The disturbances due to the presence of the wedge are confined within a curved shock, beyond which the freestream conditions prevail. In this study, we assume the shock to be a plane. For a slender wedge with a sharp leading edge, the shock is attached at the leading edge. Thus the flows on the two sides of the wedge become independent of each other and can be considered separately. Thus the problem under investigation may be reduced to the problem of finding the unsteady flow over a flat plate after an increase or decrease in its speed, and subject to the possible freestream variations in the thermodynamic properties.

Problem Formulation

A slender infinite wedge is subjected to a motion in which every point of the wedge has the same velocity vector parallel

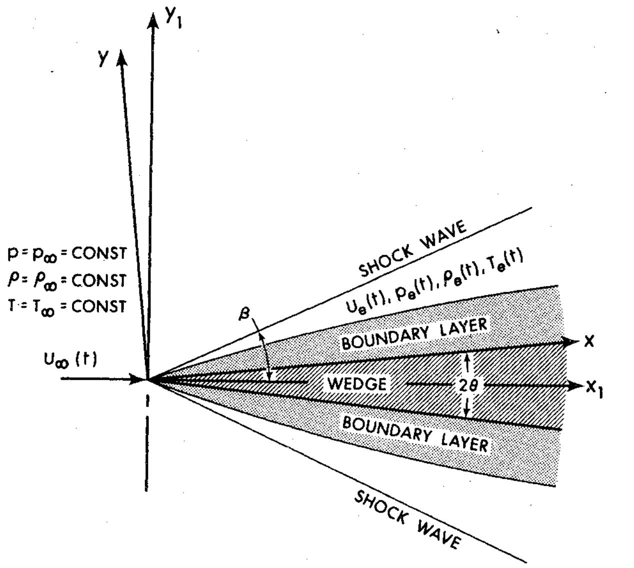

Fig. 1 Schematic of the flow configuration.

to the plane of symmetry of the wedge. Initially the wedge velocity is constant, $U = U(t_0)$. At time t_0 the wedge begins, let us say, to accelerate and reaches a new constant velocity, $U = U(t_1)$, at time t_1. During the time interval $t_0 \le t \le t_1$, the wedge velocity $U(t)$ is some function of time.

The flowfield between the shock and the boundary layer, in our formulation, is potential. In spite of this somewhat simplified configuration, even the potential part of the flow is difficult to solve analytically. We assume further that outside the boundary layer the flow is in a quasisteady state. This assumption is useful, provided that $U'(t)$ is not too large, $U(t)$ is large, and when the distance x from the edge of the wedge is small enough (primes indicate derivatives).

Some of the flow parameters are indicated in Fig. 1. Having $U_\infty(t)$, pressure p_∞, density ρ_∞, and temperature T_∞, we can calculate U_e, p_e, ρ_e, and T_e by the use of Rankine-Hugoniot relations. As the velocity $U(t)$ varies with time, the position of the shock also will change with time, and our calculations will be good if the shock velocity relative to the wedge is small. In any case, there exists a neighborhood around the edge of the wedge where this condition may be satisfied to as close a degree as we desire; i.e., there always will be a region where our procedure is applicable. Having U_e, p_e, ρ_e, and T_e, we can compute the flow parameters within the boundary layer.

Rankine-Hugoniot Conditions

In order to find the flow parameters between the boundary layer and the shock wave, we used the Rankine-Hugoniot relationships, which, when simplified, yield

$$\frac{U_e}{a_\infty} = \left\{\left[\frac{(\gamma-1)\zeta+2}{(\gamma+1)\zeta}\right]^2 \zeta - \zeta + M_\infty^2\right\}^{1/2} \tag{1}$$

$$\frac{p_e'}{p_e} = \frac{2\gamma}{\gamma+1+2\gamma(\zeta-1)}\,\frac{d\zeta}{dt} \tag{2}$$

$$\frac{T_e}{T_e'} = \frac{(\gamma+1)^2\zeta+2(\gamma-1)(\zeta-1)(\gamma\zeta+1)}{2(\gamma-1)[\gamma(\zeta-1)+\gamma(\zeta+1)/\zeta]\,d\zeta/dt} \tag{3}$$

$$\zeta = M_\infty^2 \sin^2\beta \tag{4}$$

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*Professor.

†Post-Doctoral Fellow on leave from the University of Warsaw, Poland.

$$\frac{U_e}{U_e'}=\frac{[\frac{(\gamma-1)\zeta+2}{(\gamma+1)\zeta}]^2\zeta-\zeta+M_\infty^2}{[\frac{2M_\infty}{2M_\infty\sin^2\beta-\sin2\beta}+\frac{4(1+\gamma\zeta^2)}{(\gamma+1)^2\zeta^2}]\frac{d\zeta}{dt}} \tag{5}$$

$$\beta=\frac{(\gamma+1)M_\infty^2\theta}{4(M_\infty^2-1)}+\frac{1}{M_\infty} \tag{6}$$

$$\frac{1}{a_\infty}\frac{dU_e}{dt}\approx\frac{\left[\frac{2M_\infty}{2M_\infty\sin^2\beta-\sin2\beta}-\frac{4(1+\gamma\zeta^2)}{(\gamma+1)^2\zeta^2}\right]\frac{d\zeta}{dt}}{\{[\frac{(\gamma-1)\zeta+2}{(\gamma+1)\zeta}]^2\zeta-\zeta+M_\infty^2\}^{1/2}} \tag{7}$$

where $M_\infty=U_\infty/a_\infty$ (M=Mach number, a=velocity of sound), and $a_\infty^2=\gamma p_\infty/\rho_\infty$ (specific heats ratio γ for air = 1.4).

Basic Equations

We adopt the two-dimensional boundary-layer equations. Assuming a linear absolute viscosity-temperature relationship (μ,T), constant specific heat c_p, constant but arbitrary Prandtl number Pr, and introducing a stream function ψ such that the velocity component in the x direction is given by $u=\partial\psi/\partial y$, these boundary-layer equations in terms of the Dorodnitsyn-Howarth coordinates become

$$\frac{\partial^2\psi}{\partial Y\partial t}+\frac{\partial\psi}{\partial Y}\frac{\partial^2\psi}{\partial x\partial Y}-\frac{\partial^2\psi}{\partial Y^2}\frac{\partial\psi}{\partial x}=\frac{dU_e}{dt}+\bar{\nu}\frac{\partial^3\psi}{\partial Y^3} \tag{8}$$

$$\frac{\partial T}{\partial t}+\frac{\partial\psi}{\partial Y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial Y}-\frac{\gamma-1}{\gamma}\frac{1}{p_e}\frac{dp_e}{dt}T$$

$$=\frac{\bar{\nu}}{Pr}\frac{\partial^2T}{\partial Y^2}+\frac{\bar{\nu}}{c_p}\left(\frac{\partial^2\psi}{\partial Y^2}\right)^2 \tag{9}$$

where $\bar{\nu}=\nu_\infty p_e/p_\infty$ (ν is the kinematic viscosity). Equation (8) is uncoupled from Eq. (9) and can be solved separately.

Following Moore,[1] we assume the following relationship for the dimensionless stream function:

$$\psi=(2\bar{\nu}U_ex)^{1/2}f(\eta,\xi_0,\xi_1,\ldots) \tag{10}$$

where

$$\eta=Y\left(\frac{U_e}{2\bar{\nu}x}\right)^{1/2},\quad \xi_0=\frac{xU_e'}{U_e^2},\quad \xi_1=\frac{x^2U_e''}{U_e^3},\ldots \tag{11}$$

For large U_e and moderate $U_e^{(i)}$ and x (superscript i indicates ith derivative), the quantities ξ_i may be regarded as small parameters, and, consequently, the function f may be replaced by its linear part, namely,

$$\psi=(2\bar{\nu}U_ex)^{1/2}\left[F(\eta)+\sum_{i=0}^{\infty}\xi_if_i(\eta)\right] \tag{12}$$

Similarly, we may let

$$T=T_e(t)\left[R(\eta)+\sum_{i=0}^{\infty}\xi_ir_i(\eta)\right] \tag{13}$$

Substituting Eqs. (12) and (13), respectively, into Eqs. (8) and (9) and comparing terms, we obtain a system of simultaneous ordinary differential equations for the functions $F, f_0, f_1,\ldots, R, r_0, r_1,\ldots$, the first two of which, respectively, are

$$F'''+FF''=0 \tag{14a}$$

$$f_0'''+Ff_0''-2F'f_0'+3F''f_0+2-2F'-\eta(1-\alpha)F''=0 \tag{14b}$$

...

$$R''+PrFR'+\kappa(F'')^2=0 \tag{15a}$$

$$r_0''+PrFr_0'-2PrF'r_0-2Pr\lambda^{-1}R-Pr(1-\alpha)\eta R'$$
$$+3Prf_0R'+2\frac{\gamma-1}{\gamma}\alpha PrR+2\kappa F''f_0''=0 \tag{15b}$$

...

where

$$\lambda=\frac{U_eT_e'}{U_e'T_e},\quad \delta=\frac{T_e'}{U_e'},\quad \alpha=\frac{U_e\bar{\nu}'}{U_e'\bar{\nu}},\quad \kappa=\frac{2PrU_e}{\lambda\delta c_p} \tag{16}$$

All computations were made for $Pr=0.72$.

The associated boundary conditions are

$$\psi|_{t=t_0}=(2\nu_\infty U_\infty x)^{1/2}F(\eta_0);\quad F(0)=F'(0)=0,$$
$$\lim_{\eta\to\infty}F'=1 \tag{17a}$$

$$f_0'(0)=f_0(0)=0,\quad \lim_{\eta\to\infty}f_0'=0,\ldots \tag{17b}$$

$$T|_{t=t_0}=T_\infty R(\eta_0);\quad \lim_{\eta\to\infty}R=1,\quad R'(0)=0,\ldots \tag{18}$$

where $F(\eta_0)$ and $R(\eta_0)$ are the solutions to Eqs. (8) and (9), specialized to steady state at time $t\le t_0$. Solutions to Eqs. (14) and (15a) with the respective boundary conditions were found numerically and are shown in Fig. 2. Having $F(\eta)$ and $f_0(\eta)$, we can find the approximate solution for the stream function, namely,

$$\psi=(2\bar{\nu}U_ex)^{1/2}[F(\eta)+(xU_e'/U_e^2)f_0(\eta)] \tag{19}$$

which yields the x component of velocity given by

$$u=\frac{\partial\psi}{\partial Y}=U_e\left[F'(\eta)+\frac{xU_e'}{U_e^2}f_0'(\eta)\right] \tag{20}$$

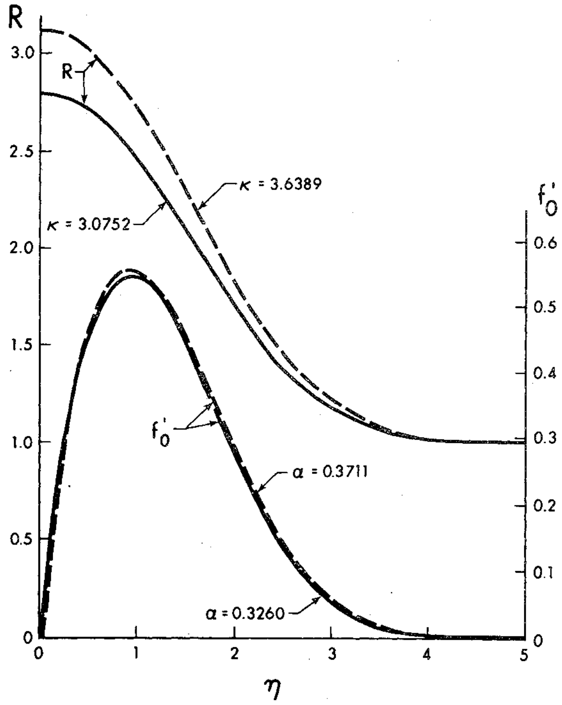

Fig. 2 Velocity coefficient f_0' and temperature coefficient R.

The expression used for the temperature distribution was $T = T_e R(\eta)$.

Equation (14a) for F is the well-known Blasius equation, and the differential equation for f_0 is of the third order but linear. In the latter, the coefficient α appears. It depends weakly on time, especially in the hypersonic flow. For example, if $8 \leq M_\infty \leq 10$, then $0.3260 \leq \alpha \leq 0.3711$ for $\theta = 5°$, $0.4295 \leq \alpha \leq 0.4792$ for $\theta = 8°$, and $0.4812 \leq \alpha \leq 0.5315$ for $\theta = 10°$. It is important that these variations of α do not cause large fluctuations in the solution for f_0. The dependence of f_0' on η can be seen in Fig. 2, where the variations in f_0' associated with a change in α also are shown. Continuous and broken lines represent solutions for $\alpha = 0.3260$ and $\alpha = 0.3711$, respectively.

Another coefficient appears in Eqs. (15), namely, κ. The first approximation of the temperature distribution depends on this coefficient. For $8 \leq M_\infty \leq 10$ and for $\theta = 5°$, the value of κ changes from 3.0752 to 3.6389. The corresponding extreme distributions of the R function are shown in Fig. 2. Significant variations in the value of R, of approximately 10%, occur near the wall. Furthermore, the refinement of the solution by the second approximation, which bears the same coefficient, appeared to be relatively insignificant. Consequently, it was decided to terminate calculation of the temperature with the first approximation.

Utilization of Solutions

The shear stress at the wall is given by

$$\tau_w = \frac{c\, p_e\, U_e}{(c_p - c_v)\rho_\infty} \left(\frac{U_e}{2\bar{\nu}x}\right)^{1/2} \left[F''(0) + \sum_{i=0}^{\infty} f_i''(0)\xi_i\right] \tag{21}$$

where c is the constant relating absolute viscosity with the temperature $\mu = ct$. For the second approximation,

$$\tau_w = \frac{c\, p_e U_e}{(c_p - c_v)\rho_\infty} \left(\frac{U_e}{2\bar{\nu}x}\right)^{1/2} \left[F''(0) + \frac{xU_e'}{U_e^2} f_0''(0)\right] \tag{22}$$

where $F''(0) = 0.4696$ and $1.3376 \leq f_0''(0) \leq 1.3566$.

Numerical Procedure

To solve Eqs. (14) with the associated boundary conditions, the Runge-Kutta method has been used. The step size was assumed to be 0.01, and the limits of integration in η were $\eta = 0$ and $\eta = 20$. Solution of Eq. (15a) could be expressed by quadrature. Numerical evaluation of the integrals expressing function R was done by the use of Simpson's method. Using a procedure based on the method suggested by Moore,[1] the linear equation (14b) was solved relatively quickly. While searching for the function f_0, it was assumed that $f_0 = af_{hom} + f_{inh}$. Here f_{hom} is the solution of Eq. (14b) with the boundary conditions given by $f_{hom}(0) = 0$, $f_{hom}'(0) = 0$, and $f_{hom}''(0) = 1$; and f_{inh} is the solution of Eq. (14b) with the conditions given by $f_{inh}(0) = f_{inh}'(0) = f_{inh}''(0) = 0$. After finding these two solutions, the constant a was obtained according to $a = -\lim_{n\to\infty} f_{inh}/f_{hom}$.

Conclusions

The proposed method enables one to compute relatively quickly the parameters associated with the hypersonic flow around the wedge. The numerical solutions can be used not only for one specific problem but also for a family of problems. This is because the coefficients in the governing equations may be determined a priori from the known or assumed variations in the Mach number and in the angle θ. Velocity of the wedge enters into the solution scheme only in its final phase.

The proposed method can be used only when the parameters ξ_i $(i = 0,1,...)$ are small. In order to meet this condition, the wedge velocity $U(t)$ must be large, but its time derivative should be sufficiently small. In addition, the region of interest should be not too far from the edge of the wedge. For the regions that are further from the edge of the wedge, but in the neighborhood adjacent to the region of our solution, the governing differential equations can be solved numerically also. Here our solution serves as the boundary condition at the initial value of x.

Acknowledgments

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Shock Penetration and Lateral Pressure Gradient Effects on Transonic Viscous Interactions

G. R. Inger*

DFVLR – AVA, Göttingen, W. Germany

Introduction

IN existing interaction theories the impinging shock is usually imposed as a boundary-layer edge condition but its subsequent penetration into the layer and the corresponding lateral interaction-pressure gradient is neglected. This is reasonably accurate in laminar flows because of their well-spread-out response to even weak shocks; however, for turbulent flows the interaction is much more violent and short range and hence the shock penetration and lateral pressure gradient effects may be important. For example, Werle and Bertke[1] found that their interacting supersonic boundary-layer model (which otherwise gives consistently good results in laminar flows) severely misrepresents experimental data and exact Navier-Stokes solutions for separating turbulent flow regardless of the viscosity model, ostensibly due to its lack of account for these effects.

The present paper deals with these features for the case of transonic normal shocks interacting with nonseparating turbulent boundary-layers; although in a lower speed range without separation the results provide useful insight as to their nature and parametric dependence.

Theoretical Model

The flow consists of a known turbulent boundary-layer profile $M_0(y)$ disturbed by a weak normal shock. Our original theory[2] was a small disturbance flow treatment, giving a linearized boundary-value problem surrounding the nonlinear shock discontinuity and underlaid by a thin Lighthill viscous sublayer (Fig. 1a). This model represents the essential features of the mixed transonic character of the nonseparating normal shock/boundary-layer interaction problem *including* lateral pressure gradient effects and is amenable to analytical treatment[2] by obtaining solutions for the three regions shown in Fig. 1a.

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*Von Humboldt Visiting Senior Research Fellow, Permanent Address: Dept. of Aerospace and Ocean Engineering, Virginia Polytechnic Institute and State University, Blacksburg, Va. Associate Fellow AIAA.